

فصل ۱

طراحی تخمین گر مقاوم به حمله مرحله‌ای

۱-۱ بسط فیلتر [۹] برای N گام تاخیر

با مقایسه ماتریس‌های کوواریانس و ضریب بهره با تاخیرهای ۱، ۲ و ۳ گام، روابط را به N گام تاخیر تعمیم می‌دهیم. ماتریس‌های بهره و کوواریانس برای N نمونه زمانی تاخیر به فرم (۴-۱۰) و (۴-۱۱) بدست می‌آیند و با استفاده از سیگما

$$\begin{aligned}
 K_{k+1} = & \left[\sum_{k+1|k} A_{k+1}^T - Q_k \left(F_k^{-T} H_k^T P_{k+1}^{(1)T} + F_k^{-T} F_{k-1}^{-T} H_{k-1}^T P_{k+1}^{(2)T} \right. \right. \\
 & \left. \left. + F_k^{-T} F_{k-1}^{-T} F_{k-2}^{-T} H_{k-2}^T P_{k+1}^{(3)T} + \dots + F_k^{-T} F_{k-1}^{-T} \dots F_{k-N+1}^{-T} H_{k-N+1}^T P_{k+1}^{(N)T} \right) \right] \\
 & \times \left\{ A_{k+1} E \{ X_{k+1} X_{k+1}^T | Z_{k+1} \} B_{k+1}^T \right. \\
 & - A_{k+1} E \{ X_{k+1} | Z_{k+1} \} B_{k+1}^T \\
 & - A_{k+1} Q_k \left[F_k^{-T} H_k^T P_{k+1}^{(1)T} + F_k^{-T} F_{k-1}^{-T} H_{k-1}^T P_{k+1}^{(2)T} + F_k^{-T} F_{k-1}^{-T} F_{k-2}^{-T} H_{k-2}^T P_{k+1}^{(3)T} \right. \\
 & \left. \left. + \dots + F_k^{-T} F_{k-1}^{-T} \dots F_{k-N+1}^{-T} H_{k-N+1}^T P_{k+1}^{(N)T} \right] \right. \\
 & - B_{k+1} E \{ X_{k+1}^T | Z_{k+1} \} A_{k+1}^T \\
 & + B_{k+1} B_{k+1}^T \\
 & + P_{k+1}^{(0)} R_{k+1} P_{k+1}^{(0)T} \\
 & \left. + P_{k+1}^{(1)} R_k P_{k+1}^{(1)T} \right\}
 \end{aligned}$$

$$\begin{aligned}
& + P_{k+1}^{(N-1)} R_{k-N+2} P_{k+1}^{(N-1)T} \\
& + P_{k+1}^{(N)} R_{k-N+1} P_{k+1}^{(N)T} \\
& - \left[P_{k+1}^{(1)} H_k F_k^{-1} + P_{k+1}^{(2)} H_{k-1} F_{k-1}^{-1} F_k^{-1} + P_{k+1}^{(3)} H_{k-2} F_{k-2}^{-1} F_{k-1}^{-1} F_k^{-1} + \dots \right. \\
& \quad \left. + P_{k+1}^{(N-1)} H_{k-N+2} F_{k-N+2}^{-1} F_{k-N+3}^{-1} \dots F_k^{-1} + P_{k+1}^{(N)} H_{k-N+1} F_{k-N+1}^{-1} F_{k-N+2}^{-1} \dots F_k^{-1} \right] Q_k A_{k+1}^T \\
& + \left[P_{k+1}^{(1)} H_k F_k^{-1} + P_{k+1}^{(2)} H_{k-1} F_{k-1}^{-1} F_k^{-1} + P_{k+1}^{(3)} H_{k-2} F_{k-2}^{-1} F_{k-1}^{-1} F_k^{-1} + \dots \right. \\
& \quad \left. + P_{k+1}^{(N-1)} H_{k-N+2} F_{k-N+2}^{-1} F_{k-N+3}^{-1} \dots F_k^{-1} + P_{k+1}^{(N)} H_{k-N+1} F_{k-N+1}^{-1} F_{k-N+2}^{-1} \dots F_k^{-1} \right] Q_k \\
& \left[F_k^{-T} H_k^T P_{k+1}^{(1)} + F_k^{-T} F_{k-1}^{-T} H_{k-1}^T P_{k+1}^{(2)T} + \dots + F_k^{-T} F_{k-1}^{-T} \dots F_{k-N+2}^{-T} H_{k-N+2}^{-T} P_{k+1}^{(N-1)T} \right. \\
& \quad \left. + F_k^{-1} \dots F_{k-N+2}^{-T} F_{k-N+1}^{-T} H_{k-N+1}^T P_{k+1}^{(N)T} \right] \\
& + \left[P_{k+1}^{(2)} H_{k-1} F_{k-1}^{-1} + P_{k+1}^{(3)} H_{k-2} F_{k-2}^{-1} F_{k-1}^{-1} + \dots + P_{k+1}^{(N-1)} H_{k-N+2} F_{k-N+2}^{-1} F_{k-N+3}^{-1} \dots F_{k-1}^{-1} \right. \\
& \quad \left. + P_{k+1}^{(N)} H_{k-N+1} F_{k-N+1}^{-1} F_{k-N+2}^{-1} \dots \right] Q_{k-1} \left[P_{k+1}^{(2)} H_{k-1} F_{k-1}^{-1} + P_{k+1}^{(3)} H_{k-2} F_{k-2}^{-1} F_{k-1}^{-1} + \dots \right. \\
& \quad \left. + P_{k+1}^{(N-1)} H_{k-N+2} F_{k-N+2}^{-1} F_{k-N+3}^{-1} \dots F_{k-1}^{-1} + P_{k+1}^{(N)} H_{k-N+1} F_{k-N+1}^{-1} F_{k-N+2}^{-1} \dots \right]^T \\
& \left. + P_{k+1}^{(N)} H_{k-N+1} F_{k-N+1}^{-1} Q_{k-N+1} F_{k-N+1}^T H_{k-N+1}^{-T} P_{k+1}^{(N)T} \right\}^{-1} \tag{۱-۱}
\end{aligned}$$

$$\begin{aligned}
\sum_{k+1|k+1} &= \sum_{k+1|k} - \sum_{k+1|k} A_{k+1}^T K_{k+1}^T \\
&+ Q_k \left[F_k^{-T} H_k^T P_{k+1}^{(1)T} + F_k^{-T} F_{k-1}^{-T} H_{k-1}^T P_{k+1}^{(2)T} + \dots \right. \\
&\quad \left. + F_k^{-T} F_{k-1}^{-T} \dots F_{k-N+2}^{-T} H_{k-N+2}^T P_{k+1}^{(N-1)T} + F_k^{-T} F_{k-1}^{-T} \dots F_{k-N+1}^{-T} H_{k-N+1}^T P_{k+1}^{(N)T} \right] \\
&- K_{k+1} A \sum_{k+1|k} \\
&+ K_{k+1} A_{k+1} E \{ X_{k+1} X_{k+1}^T | Z_{k+1} \} A_{k+1}^T K_{k+1}^T \\
&- K_{k+1} A_{k+1} E \{ X_{k+1} | Z_{k+1} \} B_{k+1}^T K_{k+1}^T \\
&- K_{k+1} A_{k+1} Q_k \left[F_k^{-T} H_k^T P_{k+1}^{(1)T} + F_k^{-T} F_{k-1}^{-T} H_{k-1}^T P_{k+1}^{(2)T} + F_k^{-T} F_{k-1}^{-T} F_{k-2}^{-T} H_{k-2}^T P_{k+1}^{(3)T} + \dots \right. \\
&\quad \left. + F_k^{-T} F_{k-1}^{-T} \dots F_{k-N+1}^{-T} H_{k-N+1}^T P_{k+1}^{(N)T} \right] \\
&- K_{k+1} B_{k+1} E [X_{k+1}^T | Z_{k+1}] A_{k+1}^T K_{k+1}^T \\
&+ K_{k+1} B_{k+1} B_{k+1}^T K_{k+1}^T \\
&+ K_{k+1} P_{k+1}^{(0)} R_{k+1} P_{k+1}^{(0)T} K_{k+1}^T \\
&+ K_{k+1} P_{k+1}^{(1)} R_k P_{k+1}^{(1)T} K_{k+1}^T \\
&+ K_{k+1} P_{k+1}^{(N-1)} R_{k-N+2} P_{k+1}^{(N-1)T} K_{k+1}^T \\
&+ K_{k+1} P_{k+1}^{(N)} R_{k-N+1} P_{k+1}^{(N)T} \\
&- [F] Q_k \\
&- K_{k+1} [F] Q_k A_{k+1}^T K_{k+1}^T \\
&+ K_{k+1} [F] Q_k \left[F_k^{-T} H_k^T P_{k+1}^{(1)T} + F_k^{-T} F_{k-1}^{-T} H_{k-1}^T P_{k+1}^{(2)T} + F_k^{-T} F_{k-1}^{-T} F_{k-2}^{-T} H_{k-2}^T P_{k+1}^{(3)T} + \dots \right. \\
&\quad + F_k^{-T} \dots F_{k-N+3}^{-T} F_{k-N+2}^{-T} H_{k-N+2}^T P_{k+1}^{(N-1)T} \\
&\quad \left. + F_k^{-T} \dots F_{k-N+2}^{-T} F_{k-N+1}^{-T} H_{k-N+1}^T P_{k+1}^{(N)T} \right] K_{k+1}^T \\
&+ K_{k+1} [M] Q_k [M]^T K_{k+1}^T \\
&+ K_{k+1} P_{k+1}^{(N)} H_{k-N+1} F_{k-N+1}^{-1} Q_{k-N+1} F_{k-N+1}^{-T} H_{k-N+1}^T P_{k+1}^{(N)T} K_{k+1}^T
\end{aligned} \tag{۲-۱}$$